

The Hidden Dimension in Maths Teaching and Learning Processes

Exploring Entrenched Beliefs, Tacit Knowledge, and Ritual Behaviour via Metaphors

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***Abstract** The paper reports some ideas and reflections concerning a study in progress which combines approaches of ethnography and cognitive linguistics. It is aimed at exploring the role that subconscious dimensions of beliefs, knowledge, and behaviour play in maths teaching and learning processes.*

Some remarks on myths, rituals, and belief systems

Human beings today are mostly seen as an essentially rational species, purposively and reasonably playing in political and social contexts. Those contexts, which we usually class under the general term of ‘modern civilisation’, are thought of as being fundamentally rooted in a sound basis of reason, knowledge, and science. Modern civilisation has devoted itself to rationalism and so insists on keeping its distance from myths, rituals, or any other ‘tribal culturemes’ – which are, rather, assigned to some savage people, cultures, and eras far below the modern horizon. Commonly, our civilisation deals with those cultures in terms of picturesque and somehow dubious objects of ethnographic research.

However, if we want to understand modern civilisation, it may be advisable to do it just the other way round, i.e. taking an ethnographer’s perspective on our civilisation itself, which of course means more than simply using ethnographic methods. From this perspective, the described confident self-concept may easily appear as a veritable myth of modern civilisation. And what should be the role of prominent modern theories, such as relativity theory and psycho-analysis, for those unable to verify or even understand them – if it is not the role of ‘grand narratives’ and myths referring to a transcendent, numinous reality? This may also apply to that

inventive system which, in a long process of career, rose from a successful instrument to the actual ‘language of knowledge’ – mathematics.

As we learned from cognitive linguistics (cf. Lakoff & Johnson 1980), our thinking, feeling and acting is fundamentally ruled by conceptual metaphors (which are more than ‘stylistic devices’ or ‘figures of speech’). So we may describe human beings as a *metaphorical species*, rather than a rational one; the rational-nonrational-dichotomy seems not adequate in this context. As thinking is playing with and within language, which is a symbolic system based upon certain rules and whose semantics is embodied in metaphors, we may understand behaviour as playing with and within a system of symbolic acts whose semantics is embodied in rituals. Rituals are symbolic or ‘metaphorised’ actions. So the metaphorical species is a *ritual species*, too, and there is in fact more ritual behaviour in all subcultures of modern civilisation than folklore has it. Even mathematics, the incarnation of rationalism, is no exception to this, neither regarding the behaviour of people being inside nor outside the mathematical community.

Most outside people’s intricate relationship with mathematics is characterised by a significant coincidence of a high esteem on the one hand and feelings of inferiority and fear on the other, of being willing to believe in what one feels unable to understand. This has a striking, and not accidental, parallel to the religious behaviour within tribal cultures described by ethnographers. It seems no exaggeration to describe, and understand, the school life of a poor maths student from the perspective of a person living in a religious tribal culture – they both experience a world divided into two realms, the ‘profane’ (*every day life*) and the ‘sacred’ (*maths*); certain rituals are to be conducted, being rules for proper behaviour in the sacred realm referring to a transcendent reality of ‘eternal verities’ and ‘ultimate values’. There are positive rituals (*you must put solutions into set brackets*) as well as negative ones, i.e. rituals of avoidance or ‘taboos’ (*you must not divide by zero*); there are rituals of transition or initiation that will allow the passage from one state to a higher one (*classroom tests, examinations*); and there are sacred places (*blackboard*), sacred times (*lessons*), and sacred objects or ‘totems’ (*p*), ruled by a sacred king, priest, or shaman (*teacher*) having admonitory visions of a ‘next life’ (*later, you will badly need maths*). Must we mention that rituals often will only be successful if entailing a sacrifice?

Though school culture in general is governed by rituals (cf. Tobin et al. 1989), there is no subject ritualised to such a high degree as maths. If we want to have maths in the students’ every day life, i.e. in the profane realm, it seems that it will not suffice to make it easier or more convenient to them nor to refine our didactic methods by some further steps. We should, rather, find ways to de-ritualise maths, which also means to de-mythologize it. Rituals and myths are closely connected with each other and with belief systems, as rituals always refer to myths which on the other hand are dependent upon belief systems. Considering their sociopsychological function, we may depict *rituals* as mise-en-scenes after scripts named *myths* which are grand narratives, in epic breadth exemplifying certain *belief systems*.

Rituals, myths, and belief systems seem to be universal features of all cultures, sometimes forming a ‘hidden dimension’ of those cultures. As to modern civilisation, there is a hidden dimension in it, too, a ‘culture behind the culture’, and there is a hidden dimension in the culture of doing, using, teaching, and learning mathematics as well. We can try to disclose it – because we can try to explore the ritual dimension of students’ and teachers’ actions by applying the ethnographic method and because we can try to explore the metaphorical dimension of their language and narratives by applying cognitive linguistics. Putting together both approaches seems to be promising for a better understanding of modern school cultures in general and especially of the ‘hidden maths culture’.

Apart from ‘sociocultural hiddenness’, which is a global and interpersonal feature, there are also individual and *intrapersonal* forms of hiddenness (or entrenchment), as human thinking, feeling, and acting always have conscious as well as subconscious (or at least entrenched) dimensions.

Some remarks on entrenched beliefs and tacit knowledge

Beliefs are phenomena that withdraw from direct observation. They can only be unfolded by the researcher in a complex and thorough process of inference and interpretation. Belief research cannot be done in the style of an opinion poll, which would be too a superficial approach – belief research is mainly interested in *deep* phenomena (cf. Ponte et al. 1999).

Regarding the degree of nearness to a person’s self, Rokeach (1979) distinguishes between *central* and *peripheral* beliefs. Inherent in central beliefs is a considerable inertia against change, and that is what makes them relevant to educational research. Improving education always implies a process of changing, which finally also means changing of beliefs. Due to their inertia, central beliefs form one important type of beliefs, another being *subconscious* beliefs. Subconscious beliefs both rule a person’s behaviour in a particular strong way and render the change process difficult, as changing cannot be done without knowing of what to change. While central beliefs are important because of their important effects on their occupants, they actually are less important within the research process as long as they are conscious or *surface* beliefs. Subconscious beliefs, however, are important, and problematic, with regard to their occupants as well as to the researcher. Particularly relevant are those beliefs which are both central and subconscious (or carefully kept under lock by the individual) and which we therefore, in analogy to some phenomena in cognitive linguistics, may call *entrenched beliefs*. From my perspective, they embody some of the most important, and most deep, phenomena of belief research.

Though, there are conscious and subconscious levels not only in belief systems, but also, and in the same fundamental sense, in knowledge systems. The most prominent contribution to this aspect is the concept of implicit or *tacit knowledge* as evolved by Michael Polanyi (1966). He emphasises the important role of creative

imagination, rooted in an implicit knowledge system, for the process of intellectual creativity. Due to Polanyi, the way in which theoretical achievement takes place, its often surprising and unforeseeable coming up, reveals that this process must be grounded in more than mere rationality. The process involved cannot simply be described on the basis of deduction processes, it is, rather, to be understood as a process of *emergence*. In evolutionary theory, this term refers to the rise of a system that cannot totally be predicted or explained from antecedent conditions. Emergence is a feature of complex systems and can be understood as a phenomenon of self-organisation. The anti-Positivist position of Polanyi appeals to modern qualitative social research and so it is, if only for that reason, of interest for belief research. In addition to this methodological aspect it gains relevance as subconscious (implicit, tacit) knowledge may have effects similar to those of subconscious (entrenched) beliefs. According to an observation of Giddens (1993, p. 158), tacit knowledge may play a role in the framing of a person's perception of reality like "the discussion of Gödel's theorem in the framing of theories".

From this perspective, which only could be shortly outlined here, I suggest that the common dichotomy 'beliefs vs. knowledge' must not be confounded with the subconscious-conscious-dichotomy. There are subconscious as well as conscious aspects crucial for both beliefs and knowledge, and even for action, reflection, and emotion. There is much reason to assume that it may be promising for belief research to focus more intensively on subconscious aspects in general.

Some research assumptions on human beings

As to my 'personal research philosophy', let me for short list some of my central research assumptions about human beings. Most of them I learned from *radical constructivism*, *ethnography*, and from the work of the *Berkeley Group* around George Lakoff, conducting the Cognitive Science Program at the Institute of Human Learning at the university of Berkeley.

About perception: An individual 'sees the world' (perceives reality) in a complex process of deconstructing 'the world' into 'multiple individual realities' (selective perception). This perception process is ruled by various 'perception interests' of the individual. The perception interests are induced by various factors, including the individual's social role.

About personal constructs: The multiple individual realities are preserved by the individual in the form of personal constructs about 'the world'. Those personal constructs are relatively stabile, having an inertia force against change. Personal constructs gain stronger stability largely not as a result of a verification process (correctness, truth), but as a result of their viability (practical benefit) and by communicative reinforcement processes (social acceptance).

About metaphors: The ordinary conceptual system of human beings, in terms of which we think and act, is fundamentally metaphorical. The personal constructs of

human beings take the form of (conceptual) metaphors. Metaphors can be shaped to (organised in) various forms of different complexity, including snapshots, paradigms, narratives. There are personal as well as interpersonal (i.e. cultural, tribal) metaphors.

About rituals: There are also metaphorised actions, called rituals. Rituals, too, can take various forms up to a high degree complexity. There are personal as well as interpersonal (i.e. cultural, tribal) rituals.

About narratives: The metaphorical process works consciously and subconsciously. An individual's narratives always have an explicit conscious surface. However, they are grounded in subconscious (entrenched) conceptual metaphors. So narratives may provide a 'bypass approach' to an individual's subconscious metaphorical grounds.

About tribal culture: From an ethnographic perspective, social groups can be analysed as tribes. Culture and cultural/tribal identity are preserved in the form of cultural/tribal metaphors and rituals. So professional identity is preserved by tribal metaphors and rituals as well. The professional life of teachers (researchers etc.) is ruled by partially conscious and partially subconscious tribal metaphors.

About change: A fundamental change of an individual's behaviour requires a change of its metaphors. Changing metaphors requires knowing about metaphors.



Fig. 1: Tribal metaphor *Sisyphos*.

Sisyphos – a tribal metaphor of maths teachers

To shortly exemplify the concept of 'tribal metaphor', we may have a look at some outcomes of an empirical study in progress, being a long-term participative research (interviews and observation). Most of the interviewed teachers turned out to

customarily see and describe their own work by reference to the *metaphor of Sisyphos*. For the background of this metaphor we may turn to one of this century's most influential philosophical books. In his essay on Sisyphos, Camus (1942) analyses contemporary nihilism by taking Sisyphos as a paradigm of an absurd existence, or even of the general absurdness of all human existence. To him, man's relationship with the world is a fundamentally absurd one. The total and voluntary surrender to the absurd is the only way to negate it, the only form of resistance against the absurd. Complete failure, finally, becomes apparent to be the only possible form of success. So Sisyphos is a model for an 'honest and upright failure'. Although his only adequate reaction ought to be committing suicide, we must image Sisyphos as a happy man, Camus suggests.

Even if we do not presume that the teachers adopt the radical view of Camus, their reference to the Sisyphos metaphor may be surprising. Deeper analysis reveals, however, that they derive some sociopsychological benefits from this metaphor. (1) The metaphor provides the benefit of *sublimation* – it raises personal experiences of failure onto a universal level; it upgrades an accident to a fateful question; it transforms a personal biography into a tribal history or myth. (2) The metaphor provides the benefit of constituting '*tribality*' – it establishes a social group (tribe) in terms of a 'community of fate' and makes the individual a member of its tribal (social, professional) culture, thus providing consolation, strengthening, and shelter. (3) The metaphor provides *relief* of personal responsibility.

A research perspective on teachers

Within the scenery of the maths teaching and learning play, different actors perceive different main aspects or 'dimensions' of a teacher. We may expect individual differences as well as group specific ones. While the respective groups of students, teachers, teacher educators, and researchers may differ in the complexity of their views, they presumably will correspond in mainly viewing a teacher as an altogether 'conscious player'. As a working hypothesis focusing on the essentials, we may take the following scale of, more and more differentiated, group specific perspectives.

Students: The mainly perceived teacher dimensions are action and character. The teacher's actions are seen as being predominantly motivated by his or her character, only secondarily by the teacher's knowledge. Teachers are mainly seen as 'actors'.

Teachers: The mainly perceived teacher dimensions are action and knowledge. The teacher's actions are seen as being predominantly motivated by his or her knowledge, only secondarily by the teacher's character. Teachers are mainly seen as 'knowing actors', i.e. actors within a 'scenery of knowledge'.

Teacher educators: The mainly perceived teacher dimensions are action, knowledge, and reflection. Teachers are mainly seen as 'reflective practitioners' (cf. Schön 1983).

Researchers: The mainly perceived teacher dimensions are action, reflection, knowledge, and beliefs. Teachers are mainly seen as ‘reflective practitioners ruled by beliefs’.

To meet the needs of my own research approach, as explained above, I suggest a refined model of teacher dimensions, with the conscious domain being paralleled by a subconscious one (see Figure 2). Here, surface beliefs, explicit knowledge, reflection, and (normal) action are mirrored by entrenched beliefs, tacit knowledge, emotion, and ritual. The model, however, is only to be taken as an synoptic organising means. Thinking of teachers in terms of vector spaces is not suitable for qualitative research.

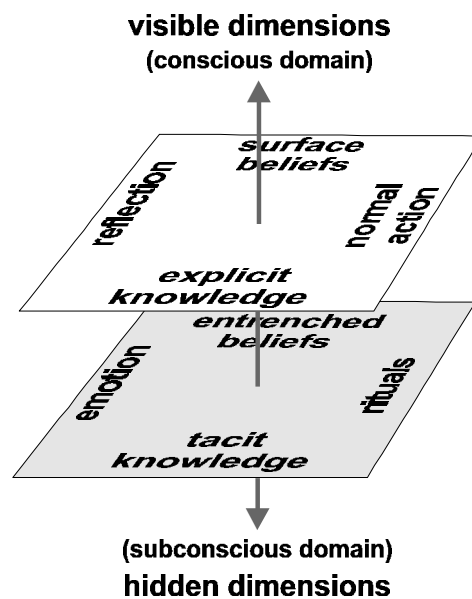


Fig. 2: Research model *Teacher Dimensions*.

To be nearer to the living world we are interested in, we may prefer to think of humans in terms of certain ‘research metaphors’ (see Figure 3). The *cobweb metaphor* views a person as being a complex natural net. Taking a discoverer’s perspective, it may be shaped to the *jungle metaphor*, depicting a person as being a jungle, i.e. a symbiotic net and living structure with mutually interdependencies, where the researcher will encounter a difficult terrain with impassable, overgrown paths and numerous promising, but nearly inaccessible locations.

A ‘bypass approach’ – why computers are interesting for belief researchers

At present, teaching, and especially maths teaching, increasingly is ‘teaching in the realm of computers’. As a previous study (cf. Berger 1998a, 1998b, 1999) has shown, a maths teacher’s computer world view has a *network of tacit links* to some of his or her beliefs concerning the nature of human thinking and learning, the nature of mathematics, and the nature of teaching and learning of maths. As the deeper layers

of a person's belief system are not directly accessible, those links provide what we may call a 'bypass approach' where the narrative about one topic (a teacher's *oral computer history*) is, largely unnoticed by the narrator, readable in another context (the nature of human thinking and of human beings).

When people describe the functions of computers, they often come up with a broad spectrum of individual aspects, yet mostly focusing on the instrumental aspects of computers. To the common view, computers are machines, tools, and mediums which are used to automate human brainwork. This depicts the function those machines actually are designed and destined for – however, this is not their only function. What makes computers interesting for belief researchers is the fact that they also have an *implicit* function. The computer serves as an objective for human projections, as a multifarious metaphor for human thinking, for the human brain, and even for a human being itself. Thus the computer is both an instrumental and a projective medium, a tool and a metaphor.

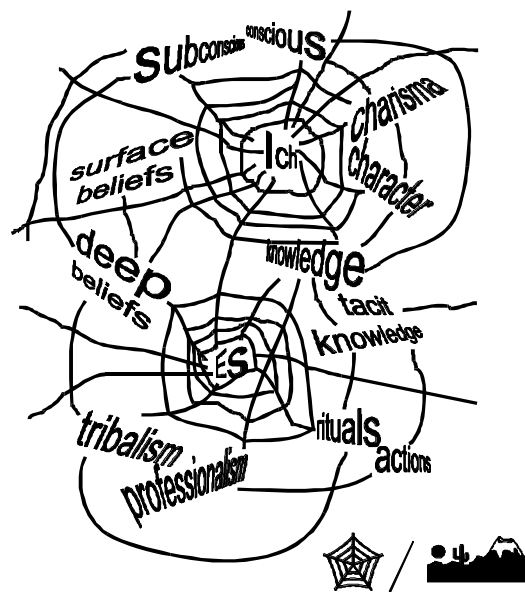


Fig. 3: Research metaphor *Cobweb / Jungle*.

New technologies always not only changed the world, but also man's thinking about this world, mostly resulting in a new and more abstract view on it. Clocks influenced man's understanding of time, steam engines changed man's view of labour, telescopes, railways, and telegraphs changed the thinking about distance. Technology always changes both human actions and human thinking. The new technology of computers, however, has an additional quality, as the new machine – in contrast to clocks, steam engines, railways, telegraphs etc. – is a 'thinking' one. It is both a copy of the human brain and a model for it. It is the first machine which changes man's thinking about thinking.

A person's (a maths teacher's) beliefs concerning computers, about their relationship to him- or herself or to other human beings, reveal a lot of his or her views of the nature of (mathematical) thinking and learning. Turkle (1984) characterises computers as man's *second self*, as 'metaphysical' and 'psychological machines', being a challenging cause for a metaphysical self-reflection of their users and having considerable psychological impact on their self-concepts and on their views of human thinking. "People are able to see themselves in the computer. The machine can seem a second self. ... In Freud's work, dreams and slips of the tongue carried the theory. Today, life on the computer screen carries theory." (Turkle 1995, p. 30 and 49).

Thus the oral computer history of a computer-experienced maths teacher is, within the framework of qualitative social research, a promising means among others to gain access to his or her entrenched beliefs and tacit knowledge concerning (not only mathematical) thinking and human beings.

Focusing on the 'hidden dimension' may, after about two decades of belief research, launch the shift from a period of a mainly descriptive belief-sampling towards a period of developing a 'grounded phenomenological beliefs theory'.

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